

de Broglie oscillation, rest mass and inertia

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Abstract

The model of a relativistic free massive point particle is investigated in the context of a Hamiltonian constraint system. It is shown that de Broglie oscillation, rest mass and inertia may be described within this model of Hamiltonian constraint system.

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1 Introduction

Newton's classical mechanics and gravitational theory underwent remarkable changes after the advent of Einstein's special and general theories of relativity. In these theories a 4-dimensional picture of our physical world was presented providing deep insight of man about the nature of physical laws. In this respect, a 4-dimensional description appeared as the key feature in investigation of the fundamental physical principles. It turned out that the 4-dimensional analysis of some physical phenomena may give the precise 3-dimensional descriptions of our experiences about them. The basic element in this 4-dimensional analysis was the abstract object called *world line element*. The invariance property of this object under Lorentz transformations or general coordinate transformations played the key role in developing special and general theories of relativity.

Regarding the key role of the world line described above it is appealing to investigate more on this object in a rather abstract way. In this respect we are looking for some global features associated with the world line which may emerge due to an invariance symmetry called *reparametrization invariance*. We pay attention to the interesting role of this symmetry in investigation of the two mysteries in modern physics, namely *de Broglie's periodic phenomenon (oscillation)* and *the inertial properties of matter*.

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Louis de Broglie had presented the central point of his hypothesis on waves of matter in a remarkable periodic phenomenon by the famous Einstein-de Broglie formula $\hbar\omega_0 = m_0c^2$, where ω_0 as the natural frequency of this phenomenon is related to the rest mass m_0 of the particle. Indeed, as is well-known, this phenomenon leads, through Lorentz transformation of the coordinate system, to de Broglie's *waves of matter*. Hence the entire quantum mechanics stands on de Broglie oscillation and any attempt to find some insight into what stands behind this phenomenon is of particular interest.

On the other hand, the issue of inertia as an *unresolved mystery in modern physics* has recently been the subject of intense investigations. Vigier [1], used the Dirac vacuum to explain, alternative to Machian viewpoint, the origin of inertia as a necessary consequence of the real particle motions described by the Einstein-de Broglie-Bohm (E.d.B.B) formalism of quantum mechanics. Recently, Rueda *et al.* in a series of papers [2] have explained the inertia as scattering-like process of the *zero point field* (ZPF) radiation subject to the electromagnetic vacuum. They showed that the scattering of the incoming ZPF flux by the fundamental particles (quarks and leptons) within the object generates a reaction force which may account, at least in part, for inertia. Moreover, the inertial mass of an object is that fraction of the energy of the ZPF radiation enclosed within the object that interacts with it. More recently, they suggested that this interaction takes place at the Compton frequency of the particle and the inertial mass of the electron is then the reaction force due to resonance scattering of the ZPF at that frequency. Furthermore, they suggested that de Broglie oscillation is due to a resonant interaction with the ZPF, hence they were able to interrelate the two above mentioned mysteries.

In this paper we take an alternative approach to study and interrelate both of two mysteries. We seek a Hamiltonian constraint structure originating from the point particle's action constructed by the *world line element*. As is well known, the action of a relativistic free massive point particle has a Hamiltonian constraint structure [3], [5] leading to the mass-shell condition, where the space-time coordinates are assumed to be as dynamical variables. Indeed, this constraint structure is independent of the space-time dimensions, so one may be allowed to reconstruct an equivalent structure without the need to use the space-time coordinates as dynamical variables. The motivation to follow this approach is to derive some valuable information about a constraint dynamics which induces the local dynamics of the point particle in space-time.

We use the *world line* of a relativistic free massive point particle as one dynamical variable and parametrize its action by a world line parameter. We then derive the associated constraint structure. This provides a framework to justify the possible origins of *de Broglie oscillation*, *rest mass* and *inertia*.

de Broglie oscillation and the rest mass may naturally emerge due to the quantum constraint associated with one first class constraint (in the terminology of Dirac) appearing in the model. Indeed, using the Einstein-de Broglie formula, this quantum constraint becomes equivalent to an eigenvalue equation whose eigenstate $e^{i\omega_0\tau}$ corresponds to de Broglie's localized oscillating field and its energy $\hbar\omega_0$ corresponds to the rest mass of the point particle (ω_0 is the Compton frequency). It is shown that what plays the role behind de Broglie oscillation and the rest mass may be this quantum constraint which is imposed on the rest point particle.

On the other hand, since the mass-shell condition as the 4-dimensional analogue of our

desired first class constraint holds at all times it leads to derivation of the relativistic equation of motion and then a force in the form $-\frac{dP_s}{d\tau}$ is interpreted physically as *inertia*. Although the physical mechanism behind this interpretation may be the ZPF scattering [2], but the *causal* origin of this reaction force seems to be in the reparametrization invariance of the point particle system which may be induced as a property resulting from the same invariance property in large universe scale. In this approach, the local inertial properties of the point particle are generated due to the “*timeless*” property of the universe and its causal structure. *Inertia of the particle then becomes a direct evidence indicating that no absolute time exists in the universe.*

2 Constraint System

In this section we will use Dirac’s formalism of Hamiltonian constraint systems [3]. We start with the action of a relativistic free massive point particle

$$I = -m_0 c \int_{s_1}^{s_2} ds \quad (1)$$

where m_0 is the rest mass of the particle, c is the velocity of light and ds is the world line element. We now parametrize the action as

$$I = -m_0 c \int_{\tau_1}^{\tau_2} \dot{s} d\tau \quad (2)$$

where $\dot{s} = \frac{ds}{d\tau}$ and τ is the world line parameter usually taken as proper time. As mentioned in the introduction, we have taken \mathbf{s} as the only dynamical variable in order to find some global characteristics of the model. The linear action (2) admits a Hamiltonian constraint structure with the primary constraint

$$\phi \equiv P_s + m_0 c \approx 0 \quad (3)$$

where P_s is the momentum conjugate to \mathbf{s} . The original Hamiltonian H_0 is zero for the linear action, so the total Hamiltonian is given

$$H_T = \lambda(\tau) \phi \quad (4)$$

where $\lambda(\tau)$ is an arbitrary function of time. Hence the dynamics is fully controlled by the constraint (3). The consistency condition

$$0 \approx \dot{\phi} = \{\phi, H_T\} \quad (5)$$

is automatically satisfied since $\{\phi, \phi\} \approx 0$, so that there is no secondary constraints. Now, the constraint (3) is by definition [3] a first class constraint which generates the following infinitesimal local gauge transformations

$$\delta \mathbf{s} = \epsilon(\tau) \{\mathbf{s}, \phi\} = \epsilon(\tau) \quad (6)$$

$$\delta P_s = \epsilon(\tau) \{P_s, \phi\} = 0 \quad (7)$$

where $\epsilon(\tau)$ is an infinitesimal arbitrary function of time. It follows from (6) that the variable \mathbf{s} has a gauge orbit. On the other hand, from (7) we find that the momentum P_s is a gauge invariant quantity on this orbit and should have a physical content. The equations of motion are

$$\dot{\mathbf{s}} = \{\mathbf{s}, H_T\} = \lambda(\tau) \quad (8)$$

$$\dot{P}_s = \{P_s, H_T\} = 0 \quad (9)$$

which integrate to

$$\mathbf{s}(\tau) = \int^\tau \lambda(\tau') d\tau' \quad (10)$$

$$P_s = \text{Const.} \quad (11)$$

These are consistent with (6) and (7) provided that $\delta\lambda = \dot{\epsilon}(\tau)$. With the identification

$$\epsilon(\tau) = \lambda(\tau)\eta(\tau)$$

where $\eta(\tau)$ is defined for an infinitesimal world line parametrization as

$$\eta(\tau) = \tau - \tilde{\tau}(\tau)$$

one can see that, using equations of motion (8) and (9), the infinitesimal local gauge transformations (6), (7) correspond to the world line reparametrizations. The freedom in the choice of Lagrange multiplier $\lambda(\tau)$ corresponds to the freedom in the choice of world line parametrization. Hence, choosing the parametrization means the gauge-fixing of the system through a choice for $\lambda(\tau)$. However, different functions $\lambda(\tau)$ leading to the same value of the action I lie on the same gauge orbit and describe identically the same physical situation.¹

3 de Broglie oscillation and rest mass

The constraint (3) as the heart of this model merits considerable attention. We know from special theory of relativity the well-known relation

$$E^2 = P^2 c^2 + m_0^2 c^4 \quad (12)$$

which relates the energy content E of a point particle to its spatial momentum P and rest mass m_0 . Using the constraint (3) $|P_s| = m_0 c$, we may rewrite (12) as²

$$E^2 = (P^2 + P_s^2) c^2 \quad (13)$$

¹ Note that as a result of equation (8) and $\delta\lambda = \dot{\epsilon}(\tau)$ with $\epsilon(\tau_i) = 0$ we have

$$\delta I = -m_0 c \delta \int_{\tau_1}^{\tau_2} \lambda(\tau) d\tau = 0.$$

Therefore, the action I is the *Teichmüller* parameter labelling the gauge orbit of the system. For positive energy with $\lambda > 0$ we have forward propagation in time for $I < 0$ (a particle) and backward propagation for $I > 0$ (an antiparticle) [5].

²The notation \approx in Eq. (3) does not mean an approximate relation. It merely tells us that the constraint $|P_s| = m_0 c$ should be imposed after derivation of the equations of motion [3].

reflecting the fact that energy content of a point particle emerges due to two types of linear momentums, its usual spatial one P and the intrinsic one P_s conjugate to the world line coordinate. For a particle at rest in space-time $P = 0$, equation (13) becomes

$$E = |P_s|c \quad (14)$$

which relates the energy content of the point particle to its intrinsic momentum P_s on the world line. This is like the corresponding formula $E = Pc$ for massless particles (photons) for which we have

$$\hbar\omega = Pc \quad (15)$$

where the angular frequency ω is related to the spatial momentum P of these particles according to *wave-particle duality*. Insisting on the generality of *wave-particle duality* we may relate, as well, an intrinsic angular frequency ω_0 (as the *wave* characteristic) to the intrinsic momentum P_s (as the *particle* characteristic) of the rest point particle as

$$\hbar\omega_0 = |P_s|c. \quad (16)$$

Therefore, using $|P_s| = m_0c$, Einstein-de Broglie relation $\hbar\omega_0 = m_0c^2$ emerges naturally due to generalization of *wave-particle duality* to the rest particles³. Einstein-de Broglie relation as realization of mass-energy equivalence relates the concept of rest mass to a type of energy which appears to be the energy of massless particles namely, $\hbar\omega_0$. Therefore, intuitively, one may think that a massive rest particle as viewed by a rest inertial observer is nothing but a photon-like particle, moving with light velocity $\lambda = c$ on its world line, whose energy content appears, through mass-energy equivalence, as the particle's rest mass⁴. This may shed light on the origin of de Broglie's oscillation $e^{-i\omega_0\tau}$ as it seems to be the time dependent part of a wave propagation along the world line associated with the photon-like particle. This is because a rest observer also moves with the light velocity $\lambda = c$ on its own world line so only the time evolving part of this wave propagation manifests.

The *wave-particle duality* for a rest particle becomes more apparent if we try to derive de Broglie oscillation from this constraint system. We remind that the entire quantum mechanics stands on *wave-particle duality* with wave functions and eigenvalues as indicating the wave and particle aspects respectively. So, if we can derive de Broglie oscillation itself from quantum mechanics then we may conclude that this periodic phenomenon follows, in principle, the *wave-particle duality*. To this end, we pay attention for quantization of the present model.

³de Broglie introduced the formula $\hbar\omega_0 = m_0c^2$ directly without mentioning to the wave-particle duality as its justification. Indeed, Einstein-de Broglie relation in the form $\hbar\omega_0 = m_0c^2$ for a rest mass does not indicate *wave-particle duality* in the sense of de Broglie's waves of matter. This is because Einstein-de Broglie relation reads as the Compton wavelength $\lambda = \frac{\hbar}{m_0c}$ and if one wants to describe it in the sense of de Broglie's waves of matter then a rest particle should have the spatial momentum $P = m_0c$ which is a contradiction. In other words, although the wavelength λ , in the left hand side, manifests the *wave* aspect but the rest mass m_0 , in the right hand side, by no means indicates the *particle* aspect. The *particle* aspect of the rest particles emerges whenever the intrinsic momentum P_s is attributed to them. This gives rise to the wave-particle duality expressed as the formula $\lambda = \frac{\hbar}{P_s}$. This formula relates the Compton wavelength λ to the particle's intrinsic momentum P_s on the world line coordinate.

⁴Here, only the mass property of the particles are compared without referring to their other properties such as spin or charge.

Quantization of this constraint system is done by operating on the Hilbert subspace $|\psi\rangle$ by the operator form of the constraint (3) as [3]

$$(\hat{P}_s + m_0 c) |\psi\rangle = 0. \quad (17)$$

Using Einstein-de Broglie relation and $i\hbar \frac{\partial}{\partial \tau} = -c \frac{\hbar}{i} \frac{\partial}{\partial \mathbf{s}}$ for the rest particle obeying $d\mathbf{s} = \lambda d\tau$ with $\lambda = c$, the quantum constraint (17) becomes the eigenvalue equation

$$i\hbar \frac{\partial}{\partial \tau} |\psi\rangle = E |\psi\rangle \quad (18)$$

with energy and wavefunction as

$$E = \hbar \omega_0$$

$$|\psi(\tau)\rangle \sim e^{-i\omega_0 \tau}. \quad (19)$$

Remarkably, the wave function (19) as the physical solution satisfying the quantum constraint (17) is exactly what we have known as de Broglie's periodic phenomenon and the eigenvalue $\hbar \omega_0$ is the corresponding energy. This wave function is the physical state describing the photon-like particle, as is observed by the rest observer. If we believe that quantum mechanics, as stands on de Broglie's oscillation, provides us the proper observables with physical reality then by translating the quantum constraint (17) into an eigenvalue equation (18) we are able to derive the rest mass m_0 , as a physical *observable* $\frac{\hbar \omega_0}{c^2}$, from de Broglie's periodic phenomenon. In other words, the rest mass m_0 itself does not qualify as an observable; it is $\frac{\hbar \omega_0}{c^2}$ which physically we observe as the rest mass [6].

Moreover, the quantum constraint (17) or equivalently Eq (18) seems to describe the quantum motion well-known as *Schrödinger Zitterbewegung*[4].

4 Inertia

The issue of inertia is basically correlated with the phenomenon of acceleration. Suppose we have a rest point particle in an inertial reference frame. Actually, since the particle does not undergo any external force the action for a free point particle (1) is then applied and leads to the first class constraint

$$\phi \equiv P_s + m_0 c \approx 0.$$

In order to find what effects such a constraint structure generates in space-time we resort to the 4-dimensional analogue of the constraint (3) as the mass-shell condition [5]

$$\psi \equiv P^\mu P_\mu + m_0^2 c^4 \approx 0 \quad (20)$$

subject to the total Hamiltonian

$$H_T = \lambda(\tau) \psi$$

where P_μ is the 4-momentum of the particle defined on the Minkowski space-time with metric $(- + + +)$. Time independence of the mass-shell condition (20) $\dot{\psi} = \{\psi, H_T\} = 0$, means trivially

$$\frac{dP_\mu}{d\tau} = 0 \quad (21)$$

which is the equation of motion for a free particle where τ is the proper time. However, we find that the following non-trivial condition

$$P^\mu \frac{dP_\mu}{d\tau} = 0 \quad (22)$$

with $\frac{dP_\mu}{d\tau} \neq 0$ may also be satisfied in consistency with the first class constraint (20). To this end, we define the generalized spatial force Q_i as the *force of constraint*

$$Q_i \equiv F_i - \frac{dP_i}{d\tau} \quad i = 1, 2, 3 \quad (23)$$

where F_i indicates the spatial components of the four-force F_μ and $-\frac{dP_i}{d\tau}$ is assumed to be a force with physical origin in the same foot as F_i . The physical nature of the force $-\frac{dP_i}{d\tau}$ may be justified according to the recent results obtained by Rueda *et.al.* in that a reaction (physical) force $\vec{F}_r = \frac{d\vec{P}_r}{d\tau} = -\frac{d\vec{P}}{d\tau}$ (generated by ZPF scattering) is interpreted as inertia so that one can rewrite equation (23) as $Q \equiv \vec{F} + \vec{F}_r$ [2]. It is easy to show that $Q_i = 0$ satisfies the condition (22). To see, we rewrite (22) as

$$E \frac{dE}{d\tau} - P_i \frac{dP_i}{d\tau} c^2 = 0 \quad (24)$$

which leads to

$$F_i = \frac{dP_i}{d\tau} \quad (25)$$

where

$$\frac{dE}{d\tau} = F_i V_i \quad , \quad E = \gamma m_0 c^2 \quad \text{and} \quad P_i = \gamma m_0 V_i$$

have been used and γ is the relativistic factor [7]⁵. Therefore, the vanishing of the force of constraint Q_i implies the mass-shell condition to be hold at all times. The interpretation of vanishing Q_i is that the acceleration, as well as uniform motion, is also consistent with this first class constraint such that the active force F_i is balanced, according to Newton's third law, by an equal and opposite directional (physical) force $-\frac{dP_i}{d\tau}$ which we shall interpret it as inertia. As far as we concern with the present constraint system it is reasonable to say that during the acceleration of particle a reaction force against the agent of acceleration is generated as the price to be paid to preserve the constraint (20) which defines, on the other hand, the causal structure of space-time in the model.

As is usual in the study of non-inertial frames in classical mechanics, we may also transfer the term $-\frac{dP_i}{d\tau}$ to the right hand side of equation $Q_i = 0$ to obtain the equation (25) which is interpreted as the relativistic equation of motion. These interpretations will be reasonable if we apply the general law of motion in the form

$$\sum \text{Physical Force} = \frac{d}{d\tau} (\text{Particle Momentum}) \quad (26)$$

in both inertial and accelerated (co-moving) frames of reference. Then the equation

$$F_i - \frac{dP_i}{d\tau} = 0 \quad (27)$$

⁵Here, the four-momentum denoted by K_μ in [7] is exchanged symbolically by F_μ for convenience.

compared to (26) implies that in the left hand side the agent of particle dynamics, namely the physical motive force F_i co-moving with the particle experiences an equal and opposite directional *physical* force $-\frac{dP_i}{d\tau}$ as the inertia which results in the right hand side a zero acceleration of particle in the co-moving frame. A realistic example for this situation is to suppose an accelerated rocket to which a particle has been attached by an string. In the co-moving frame, the rocket will then experience the inertia of particle as a reaction force against the active force imposed by string, but no acceleration is attributed to the particle simply because it moves in contact with the rocket. On the other hand, the equation written as

$$F_i = \frac{dP_i}{d\tau}$$

compared to (26) implies that the physical force F_i (in the left hand side) produces a dynamics for the particle according to $\frac{dP_i}{d\tau}$ (in the right hand side) in the inertial frame and expresses the relativistic equation of motion.

Equations (25) and (27) may be also interpreted as expressing the circular motion of the point particle in both inertial and rotating frames of reference, respectively. In this respect, F_i would be the centripetal force and the term $-\frac{dP_i}{d\tau}$ would indicate the centrifugal force in the rotating frame.

We conclude that in the context of present paper *inertia as the reaction force against acceleration may emerge due to resistance of the first class constraint to breakdown*. The breakdown occurs either when an absolute time parameter exists or when the causal structure of space-time breaks down. Therefore, it may be said that the absence of an absolute time parameter or, equivalently, the causal structure described by the mass-shell condition in this system is the possible conceptual origin of the inertia. On the other hand, we know from cosmology that the universe, as a whole, may be described by a similar constraint structure as here with a first class constraint known as Wheeler-DeWitt equation. This equation also implies that there is no absolute time parameter external to the universe.

For an interested Machian, it would be appealing to relate the whole universe and the inertia of particle through the general property of the universe: “*There is no absolute time*”. In this regard, the inertia of a particle emerging from reparametrization invariance of its action may be induced locally by this global cosmological property. This is because it is not reasonable to have no absolute time in cosmological scale but have an absolute time in local scales.

The old question that “*How a material object understands its motion to manifest the inertial effects?*” was answered by Newton as: “*It feels the action of absolute space*”, and by Mach as: “*It feels the action of cosmic matter*”. Another answer, based on this approach, may be given as: “*It knows the absence of absolute time and feels the action of space-time causal structure*”. Therefore, the universal property of reparametrization invariance seems to be a missed chain which links the universe to the local particle dynamics without the need to *action at distance* violating the causality principle. In simple words, the state of being at rest, in uniform motion and in accelerated motion of a particle are correlated with the state of universe through this universal property. In this regard, the first and second laws of Newton seem to be two different manifestations of this universal property (see equations (21) and (25)). Although this approach to the issue of inertia is not fully Machian ⁶ (as there is no direct

⁶Wheeler-DeWitt equation is also valid for an empty universe. Therefore, in the context of present paper

role playing by the matter content of the universe) but, unlike the Machian viewpoint, has the advantage of being compatible with general relativity as a realization of the reparametrization invariant theory. If correct, this approach would substitute for Mach's principle and imply that the inertial effects are pure relativistic in nature preserving the causality in general agreement with [2].

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the particle may have inertia even in an empty universe.